

Peculiar long-range supercurrent in SFS junction containing a noncollinear magnetic domain in the ferromagnetic region

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We study the supercurrent in a superconductor-ferromagnet-superconductor heterostructure containing a noncollinear magnetic domain in the ferromagnetic region. It is demonstrated that the magnetic domain can lead to a spin-flip process, which can reverse the spin orientations of the singlet Cooper pair propagating through the magnetic domain region. If the ferromagnetic layers on both sides of magnetic domain have the same features, the long-range proximity effect will take place. That is because the singlet Cooper pair will create an exact phase-cancellation effect and gets an additional π phase shift as it passes through the entire ferromagnetic region. Then the equal spin triplet pair only exists in the magnetic domain region and can not diffuse into the other two ferromagnetic layers. So the supercurrent mostly arises from the singlet Cooper pairs and the equal spin triplet pairs are not involved. This behavior is quite distinct from the common knowledge that long-range supercurrent induced by inhomogeneous ferromagnetism stems from the equal spin triplet pairs. The result we presented here provides a new way for generating the long-range supercurrent.

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The interplay between superconductivity and ferromagnetism in mesoscopic structures has been extensively studied because of the underlying rich physics and potential applications in spintronics and quantum information¹⁻⁴. When a homogeneous ferromagnet (F) is sandwiched between two s-wave superconductors (S) to form a Josephson junction, the magnetic configuration of the F layer may substantially modify the spatial properties of the superconducting order parameter. This behavior is induced by the different action of the ferromagnetic exchange field h on the spin-up and spin-down electrons that form the Cooper pair. Then this pair inside the F layer acquire a relative phase shift $Q \cdot R$, where $Q \simeq 2h/\hbar v_F$, v_F is the Fermi velocity and R is the thickness of the F layer. This phase shift changes with R and results in an oscillation of critical current accompanied with a rapid decay. The above oscillation will lead to the transition from the so-called 0 state to the π state¹⁻⁴.

Two kinds of approaches have been proposed to produce long-range supercurrent in SFS junction. The first approach requires inhomogeneous magnetism in F region so that the interested equal spin triplet pairs can be generated². One way to achieve this purpose is by arranging ferromagnetic trilayer with noncollinear magnetizations⁵. In this geometry, the spin-flip processes at the interface F layers can convert the singlet Cooper pairs into the equal spin triplet pairs^{4,6}. The triplet pairs can penetrate into the central F layer over a long distance unsuppressed by the exchange interaction so that the proximity effect is enhanced. The second approach requires the F layer to be arranged antiparallel. This situation was described by Blanter *et al.* for a $SFFS$ junction⁷. The physical origin of the enhanced proximity effect is described as a compensation of the relative phase shift of a Cooper pair as it passes from the first F layer into the second one. If the two F layers have the same thickness, the net change in

the relative phase of the Cooper pair is zero in the clean limit. This enhanced Josephson current has been proved by recent experiment⁸.

In this paper, we predict the third approach to generate the long-range supercurrent in SFS structure, shown schematically in figure 1(a). The ferromagnetic region consists of two ferromagnetic layers (F_L and F_R) with magnetizations oriented in same directions. The F_L layer and F_R layer are separated by a clean magnetic domain (F_M), whose magnetization is misaligned with direction of the F_L layer and F_R layer. The magnetic domain F_M can induce a spin-flip process, which reverses the spin orientations of the singlet Cooper pair propagating through the F_M region. This process will make the singlet Cooper pair create a phase-cancellation effect and obtain an additional π phase shift. If the F_L layer and F_R layer have the same thickness and exchange field, the net change in the relative phase of a singlet Cooper pair is π when it passes through the entire F region. In this case, the contribution to the long-range supercurrent mostly arises from the singlet Cooper pairs. This is because the equal spin triplet pairs only display in F_M region and can not diffuse into the F_L and F_R layers.

In our numerical calculation, the transport direction is along the y axis, and the system is assumed to be infinite in the x - z plane. The BCS mean-field effective Hamiltonian^{1,9} is

$$H_{eff} = \int d\vec{r} \left\{ \sum_{\alpha, \beta} \psi_{\alpha}^{\dagger}(\vec{r}) [H_e(\hat{1})_{\alpha\beta} - (\vec{h} \cdot \vec{\sigma})_{\alpha\beta}] \psi_{\beta}(\vec{r}) + \frac{1}{2} \left[\sum_{\alpha, \beta} (i\sigma_y)_{\alpha\beta} \Delta(\vec{r}) \psi_{\alpha}^{\dagger}(\vec{r}) \psi_{\beta}^{\dagger}(\vec{r}) + h.c. \right] \right\}, \quad (1)$$

where $H_e = -\hbar^2 \nabla^2 / 2m - E_F$, $\psi_{\alpha}^{\dagger}(\vec{r})$ and $\psi_{\alpha}(\vec{r})$ are creation and annihilation operators with spin α . $\hat{\sigma}$ and E_F are Pauli matrices and the Fermi energy, respec-

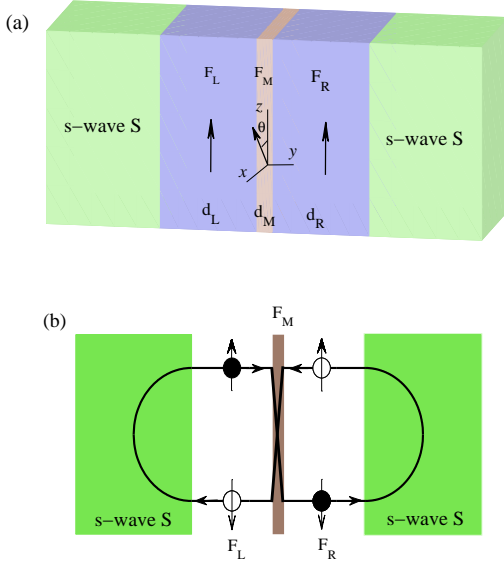


FIG. 1. (color online) (a) Schematic diagram of SF/S structure with two ferromagnetic layers F_L and F_R oriented along the z axis and separated by a noncollinear magnetic domain F_M . The lengths of F_L , F_R and F_M are denoted by d_L , d_R and d_M , respectively. The phase difference between the two S s is $\phi = \phi_R - \phi_L$. (b) The transmission of electron and hole in above structure.

tively. The superconducting gap is given by $\Delta(\vec{r}) = \Delta(T)[e^{i\phi_L}\Theta(-y) + e^{i\phi_R}\Theta(y - d_F)]$ with $d_F = d_L + d_M + d_R$. Here, $\Delta(T)$ accounts for the temperature-dependent energy gap. It satisfies the BCS relation $\Delta(T) = \Delta_0 \tanh(1.74\sqrt{T_c/T - 1})$ with T_c the superconducting critical temperature. $\Theta(y)$ is the unit step function, and $\phi_{L(R)}$ is the phase of the left (right) S . The exchange field \vec{h} due to the ferromagnetic magnetizations in the F region is described by

$$\vec{h} = \begin{cases} h_L \hat{z}, & 0 < y < d_L \\ h_M(\sin \theta \hat{x} + \cos \theta \hat{z}), & d_L < y < d_L + d_M \\ h_R \hat{z}, & d_L + d_M < y < d_F, \end{cases} \quad (2)$$

where θ is the misorientation angle of magnetization in the magnetic domain F_M region. Based on the Bogoliubov transformation $\psi_\alpha(\vec{r}) = \sum_n [u_{n\alpha}(\vec{r})\hat{\gamma}_n + v_{n\alpha}^*(\vec{r})\hat{\gamma}_n^\dagger]$ and the anticommutation relations of the quasiparticle annihilation and creation operators $\hat{\gamma}_n$ and $\hat{\gamma}_n^\dagger$, we have the Bogoliubov-de Gennes (BdG) equation^{1,9}

$$\begin{pmatrix} \hat{H}(y) & \hat{\Delta}(y) \\ -\hat{\Delta}^*(y) & -\hat{H}^*(y) \end{pmatrix} \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \end{pmatrix} = E \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \end{pmatrix}, \quad (3)$$

where $\hat{H}(y) = H_e \hat{1} - h_z(y)\hat{\sigma}_z - h_x(y)\hat{\sigma}_x$ and $\hat{\Delta}(y) = i\hat{\sigma}_y \Delta(y)$. Here $\hat{u}(y) = (u_\uparrow(y), u_\downarrow(y))^T$ and $\hat{v}(y) = (v_\uparrow(y), v_\downarrow(y))^T$ are two-component wave functions.

Since the transversal momentum components are conserved, we consider the configuration in the one dimensional regime for simplicity. The BdG equation can be

easily solved for each superconducting electrode and each F region, respectively. The scattering problem can be solved by considering the boundary conditions at the interfaces. Each interface gives a scattering matrix. The total scattering matrix of the system can be obtained by the combination of all these scattering matrices of the interfaces. From the total scattering matrix, we can obtain the Andreev reflection amplitudes $a_1 \sim a_4$ of the junction where $a_{1(2)}$ is for the reflection from an electron-like to a hole-like quasiparticle with spin up (down), and $a_{3(4)}$ is for the reverse process with spin up (down)^{10,11}. The dc Josephson current can be expressed in terms of the Andreev reflection amplitudes by using the finite-temperature Green's function formalism¹²⁻¹⁵

$$I_e(\phi) = \frac{k_B T e \Delta}{4\hbar} \sum_{k_\parallel} \sum_{\omega_n} \frac{k_e(\omega_n) + k_h(\omega_n)}{\Omega_n} \left[\frac{a_1(\omega_n, \phi) - a_2(\omega_n, \phi)}{k_e} + \frac{a_3(\omega_n, \phi) - a_4(\omega_n, \phi)}{k_h} \right], \quad (4)$$

where $\omega_n = \pi k_B T (2n + 1)$ are the Matsubara frequencies with $n = 0, 1, 2, \dots$ and $\Omega_n = \sqrt{\omega_n^2 + \Delta^2(T)}$. $k_e(\omega_n)$, $k_h(\omega_n)$, and $a_j(\omega_n, \phi)$ with $j = 1, 2, 3, 4$ are obtained from k_e , k_h , and a_j by analytic continuation $E \rightarrow i\omega_n$. $k_{e(h)}$ is the wave vector for electron (hole) in the S s. Then the critical current is derived from $I_c = \max_\phi |I_e(\phi)|$.

In principle, to obtain the spin singlet pair and the spin triplet pair amplitude functions as well as the local density of the states (LDOS), we need to solve the BdG equation (3) by Bogoliubov's self-consistent field method^{9,16,17}. We put the junction in a one dimensional square potential well with infinitely high walls. Accordingly, the solution in equation (3) can be expanded in terms of a set of basis vectors of the stationary states¹⁸, $u_\alpha(\vec{r}) = \sum_q u_q^\alpha \zeta_q(y)$ and $v_\alpha(\vec{r}) = \sum_q v_q^\alpha \zeta_q(y)$ with $\zeta_q(y) = \sqrt{2/d} \sin(q\pi y/d)$. Here q is a positive integer and $d = d_{S1} + d_F + d_{S2}$. d_{S1} and d_{S2} are the length of the left and right S s, respectively. The BdG equation (3) is solved iteratively together with the self-consistency condition $\Delta(y) = g(y)f_3(y)$ ⁹. Here the singlet pair amplitude is given by¹⁷

$$f_3(y) = \frac{1}{2} \sum_{0 \leq E \leq \omega_D} [u_\uparrow(y)v_\downarrow^*(y) - u_\downarrow(y)v_\uparrow^*(y)] \tanh\left(\frac{E}{2k_B T}\right), \quad (5)$$

where ω_D is the Debye cutoff energy. The effective attractive coupling $g(y)$ will be taken to be zero outside the S and a constant within it. Iterations are performed until self-consistency is reached, starting from the stepwise approximation for the pair potential. The spin triplet pair with zero spin projection and the equal spin triplet pair amplitude functions are defined as follows¹⁷

$$f_0(y, t) = \frac{1}{2} \sum_{E > 0} [u_\uparrow(y)v_\downarrow^*(y) + u_\downarrow(y)v_\uparrow^*(y)] \eta(t), \quad (6)$$

$$f_1(y, t) = \frac{1}{2} \sum_{E > 0} [u_\uparrow(y)v_\uparrow^*(y) - u_\downarrow(y)v_\downarrow^*(y)] \eta(t), \quad (7)$$

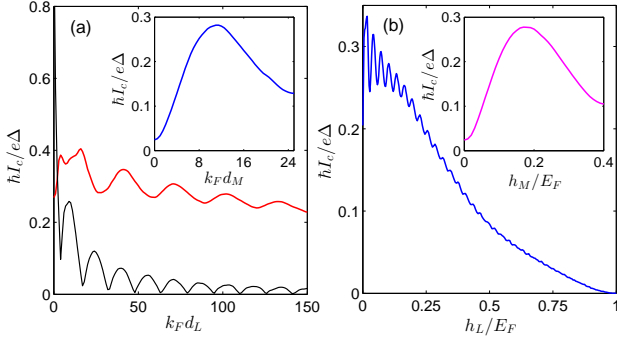


FIG. 2. (Color online) (a) Critical current as a function of length $k_F d_L (= k_F d_R)$ for exchange field $h_M/E_F = 0$ (black line) and $h_M/E_F = 0.17$ (magenta line) then $k_F d_M = 10$, and inset shows the critical current versus $k_F d_M$ for $k_F d_L = k_F d_R = 100$. Parameters used in (a): $h_L/E_F = h_R/E_F = 0.1$ and $h_M/E_F = 0.17$. (b) Critical current as a function of exchange field $h_L/E_F (= h_R/E_F)$ for $h_M/E_F = 0.17$, and inset shows the critical current versus h_M/E_F for $h_L/E_F = h_R/E_F = 0.1$. Parameters used in (b): $k_F d_L = k_F d_R = 100$ and $k_F d_M = 10$. In all plots $\theta = \pi/2$.

where $\eta(t) = \cos(Et) - i \sin(Et) \tanh(E/2k_B T)$. The above singlet and triplet pair amplitudes are all normalized to the value of the singlet pairing amplitude in a bulk S material. The LDOS is given by¹⁷

$$N(y, \epsilon) = - \sum_{\alpha} [u_{\alpha}^2(y) f'(\epsilon - E) + v_{\alpha}^2(y) f'(\epsilon + E)], \quad (8)$$

where $f'(\epsilon) = \partial f / \partial \epsilon$ is the derivative of the Fermi function. The LDOS is normalized by its value at $\epsilon = 3\Delta_0$ beyond which the LDOS is almost constant.

Unless otherwise stated, we use the superconducting gap Δ_0 as the unit of energy. The Fermi energy is $E_F = 1000\Delta_0$ and temperature is $T/T_c = 0.1$. In self-consistent field method, we consider the low temperature limit and take $k_F d_{S1} = k_F d_{S2} = 400$, $\omega_D/E_F = 0.1$, the other parameters are the same as the ones mentioned above.

In figure 2(a), we present the dependence of the critical current I_c on the length $k_F d_L (= k_F d_R)$ for two different exchange fields of F_M region. It is well known that, if $h_M/E_F = 0$ the spin-flip process does not occur in F_M region, the critical current I_c exhibits oscillations with a period $2\pi\xi_F$ and simultaneously decays exponentially on the length scale of ξ_F ¹. Here, ξ_F is the magnetic coherence length. The main reason is described below: a Cooper pair entering into the F layer receives a finite momentum Q from the spin splitting of up and down bands. So the spin singlet pairing state $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ in S will be converted into the mixed state $|\uparrow\downarrow\rangle e^{iQ \cdot R} - |\downarrow\uparrow\rangle e^{-iQ \cdot R}$ in F layer, thus leading to a modulation of the pair amplitude with the thickness R of F layer. Then the $0 - \pi$ transition will arise due to spatial oscillations of the pair amplitude. In this case the phase shift induced by the F layer is additive and the relative phase is generally nonzero so that the supercurrents are suppressed.

In contrast, when $h_M/E_F = 0.17$, I_c will slowly decrease with the thickness $k_F d_L$. The electron and hole transport process is shown in figure 1(b). Because the magnetization direction of the F_M region is along the x axis ($\theta = \pi/2$), which is orthogonal to the magnetization in F_L layer and F_R layer, the spin-flip process will appear in the F_M region. As a result, when a electron $|\uparrow\rangle_e$ transmits from F_L layer to F_R layer, the spin-flip can convert $|\uparrow\rangle_e$ into $|\downarrow\rangle_e$. Subsequently, the $|\downarrow\rangle_e$ is Andreev reflected at $F_R S$ interface and will be further converted into hole $|\uparrow\rangle_h$. The $|\uparrow\rangle_h$ moving to left is consequently inverted to $|\downarrow\rangle_h$. Finally, this $|\downarrow\rangle_h$ will propagate to the $S F_L$ interface and be reflected back as the original $|\uparrow\rangle_e$. Hence, the spin-flip process occurring in the F_M region can reverse the spin orientations of the electron and Andreev-reflected hole transporting between F_L layer and F_R layer. While if the mixed state $|\uparrow\downarrow\rangle e^{iQ \cdot R} - |\downarrow\uparrow\rangle e^{-iQ \cdot R}$ passes from F_L layer into F_R layer, it will be converted into a new state:

$$\begin{aligned} & |\uparrow\downarrow\rangle e^{iQ' \cdot R'} - |\downarrow\uparrow\rangle e^{-iQ' \cdot R'} \\ &= -(|\uparrow\downarrow\rangle e^{-iQ' \cdot R'} - |\downarrow\uparrow\rangle e^{iQ' \cdot R'}) \quad (9) \\ &= |\uparrow\downarrow\rangle e^{i(-Q' \cdot R' + \pi)} - |\downarrow\uparrow\rangle e^{-i(-Q' \cdot R' + \pi)}. \end{aligned}$$

Therefore, when the Cooper pair passes through the F_L layer, it will acquire a relative phase shift $\delta\chi_1 = Qd_L$. Similarly, traversing through the F_R layer, it gets the other phase shift $\delta\chi_2 = -Q'd_R + \pi$. As a result, this situation can be described as a superposition of the phase shift of the Cooper pair as it travels through the entire F region: $\chi = \delta\chi_1 + \delta\chi_2$. If the F_L and F_R layer have the same exchange field and thickness, the net change in the relative phase of the Cooper pair is $\chi = \pi$, and it will not turn to 0 with increase of the length of F_L layer and F_R layer. Provided one does not take absolute value for $I_e(\phi)$ to define the critical current I_c , I_c is always negative and is correspond to the π state. As the magenta line shown in figure 2(a), I_c also displays an oscillatory behavior with increasing the length $k_F d_L$. From this behavior, it is easier to deduce that the spin orientations of the part of Cooper pairs will be inverted by spin-flip in F_M region, so that these pairs can lead to a substantially enhanced critical current but do not provide the oscillations for this current. However, the rest of Cooper pairs pass through the F_M region without spin-flip and can not generate the cancellation of the relative phase. Consequently, the transmission of these pairs makes the critical current oscillate with the length of entire F region. In addition, another interesting property is the nonmonotonic dependence of the critical current I_c as a function of the length $k_F d_M$ (see the inset in figure 2(a)). We find that the maximum of I_c is nearly located at $k_F d_M = 10$. It means that the spin-flip ratio reaches their maximal value in this condition.

Next, we discuss the dependence of critical current I_c on the exchange field $h_L/E_F (= h_R/E_F)$. As plotted in figure 2(b), I_c almost decreases monotonically to 0 with the increase of h_L/E_F . This is easily understood, because normal Andreev reflection occurred at

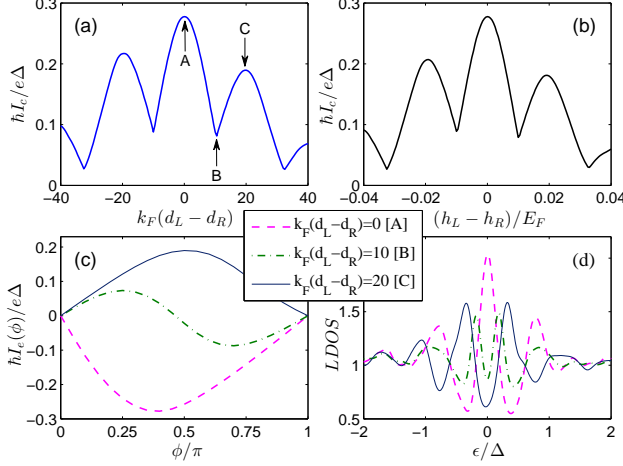


FIG. 3. (Color online) Critical current (a) as a function of the length difference $k_F(d_L - d_R)$ for $h_L/E_F = 0.1$, and (b) as a function of the exchange field difference $(h_L - h_R)/E_F$ for $k_F d_L = 100$. The current-phase relation $I_c(\phi)$ (c) and the averaged LDOS in all F region (d) corresponding to the point A, B and C in panel (a). The parameters in F_M and F_R layers have the fixed values $k_F d_M = 10$, $k_F d_R = 100$, $h_M/E_F = 0.17$, $h_R/E_F = 0.1$ and $\theta = \pi/2$. Here the LDOS is calculated at $k_B T = 0.001$.

the SF_L and $F_R S$ interface will be suppressed by exchange splitting of F_L layer and F_R layer. Especially, if $h_L/E_F = h_R/E_F = 1$, the F_L layer and F_R layer are all converted into half metal and only one spin band can be occupied, then the Andreev reflections at the interfaces will be completely prohibited. In this case, none of Cooper pairs can transmit from the left S to the right one, so the Josephson current would be complete suppressed. This feature further demonstrates that the current mostly arises from the contribution of the singlet Cooper pairs but not the equal spin triplet pairs. That is because the equal spin triplet pairs can penetrate over a long distances into the half metal and will scarcely be affected by exchange splitting^{6,19}. In addition, the inset in figure 2(b) shows the h_M/E_F dependence of the critical current I_c . It also displays a nonmonotonic behavior as the h_M/E_F is increased, and the maximum is nearly seated at $h_M/E_F = 0.17$.

To further demonstrate the conclusion mentioned beforehand, we now discuss intriguing influence of the length and exchange field on the critical current I_c when the F_L layer and F_R layer have nonidentical physical features. As illustrated in figure 3(a) and (b), we present the dependence of I_c on $k_F(d_L - d_R)$ and $(h_L - h_R)/E_F$ respectively on condition that the exchange field and length of F_R layer are all fixed. Take first one for example, we find that the dependence of I_c on length difference $k_F(d_L - d_R)$ look like a ‘‘Fraunhofer pattern’’. With the length difference close to 0, I_c will increase and also accompanies the transition between the 0 and π states. As mentioned above, if F_L layer and F_R layer have

the identical exchange fields ($h_L/E_F = h_R/E_F = 0.1$) but different lengths, the Cooper pair passing through the F_L layer and F_R layer could acquire the phase shift $\chi = Q(d_L - d_R) + \pi$. So the variation of length difference can lead to the oscillation of I_c . On the other hand, if F_L layer and F_R layer have the same length ($k_F d_L = k_F d_R = 100$) but different exchange fields, the Cooper pair can get the phase shift $\chi = (Q - Q')d_L + \pi$. I_c will also oscillate with $(h_L - h_R)/E_F$ because of $Q \propto h$ (see figure 3(b)).

In addition, the current-phase relations $I_c(\phi)$ in particular points are illustrated in figure 3(c), and corresponding LDOSs are plotted in figure 3(d). If one take the identical parameter in F_L and F_R layer, such as $k_F d_L = k_F d_R = 100$ and $h_L/E_F = h_R/E_F = 0.1$, the Cooper pairs will obtain a net phase shift $\chi = \pi$. Then we could observe a negative Josephson current and a sharp zero energy conductance peak in LDOS, which indicate the junction is located in π state. When the length difference $k_F(d_L - d_R) = 10$, it corresponds to the transition point between the 0 and π states of the junction. At this critical point, the harmonic $I_1 \sin \phi$ of the current ($I(\phi) = I_1 \sin \phi + I_2 \sin(2\phi) + \dots$) vanishes, and the $I_2 \sin(2\phi)$ will be fully revealed. Subsequently, the sign of I_c is changed with the increase of the length difference. For $k_F(d_L - d_R) = 20$, the junction is in the 0 state, and the LDOS at $\epsilon = 0$ will be converted from the peak to a valley.

Finally, we discuss the dependence of I_c on the misorientation angle θ , and the spatial distributions of the spin singlet pair and the spin triplet pair for two different θ . One can see from the inset in figure 4(a) that, when the orientation of the magnetization in the F_M region is perpendicular to the direction of F_L layer and F_R layer, the critical current reaches the maximum. However, it decreases to minimum on condition that the magnetization of F_M region is parallel or antiparallel to the one in F_L layer and F_R layer. For given thickness of the F_M region, it is possible to find the exchange field at which switching between parallel and perpendicular orientations will lead to switching of I_c from near-zero to a finite value. This effect may be used for engineering cryoelectronic devices manipulating spin-polarized electrons. Furthermore, we find the spin singlet pair amplitude f_3 oscillates in all F region at $\theta = 0$. But it will be coherent counteracted at $\theta = \pi$ (see the main plot in figure 4(a)). In contrast, the spin triplet pair amplitude f_0 are almost identical in above two cases. For $\theta = 0$, the equal spin triplet pair amplitude f_1 is zero in entire F region. However, f_1 only survives in F_M region and can not exist in F_L layer and F_R layer under the condition of $\theta = \pi$. From these consequences, we can derived that the long-range supercurrent mostly arises from the coherent propagation of f_3 , and the contributions of f_0 and f_1 can be ruled out. To further uncover this contribution, we take into account f_3 induced by only one S . According to above theory, the spin mixed state state in F_L layer is expressed as $|\uparrow\downarrow\rangle e^{iQ \cdot R} - |\downarrow\uparrow\rangle e^{-iQ \cdot R} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \cos(Q \cdot R) + i(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \sin(Q \cdot R)$.

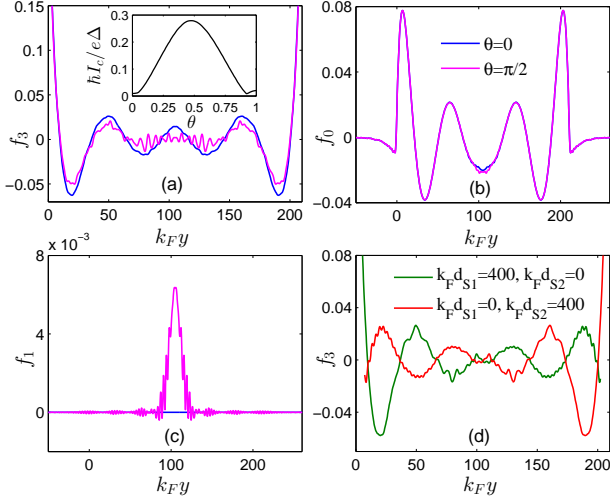


FIG. 4. (color online) Spatial distributions of the spin singlet pair amplitude f_3 (a), the real parts of spin triplet pair amplitude f_0 (b) and f_1 (c) for two angles $\theta = 0$ and $\theta = \pi/2$ at $\omega_D t = 12$. The panels (a), (b) and (c) utilize the same legend. Inset in (a) shows the critical current as a function of the angle θ . (d) The f_3 plotted as a function of $k_F y$ for two cases $k_F d_{S1} = 400$, $k_F d_{S2} = 0$ and $k_F d_{S1} = 0$, $k_F d_{S2} = 400$ when $\theta = \pi/2$. Parameters used in all figures: $k_F d_L = k_F d_R = 100$, $k_F d_M = 10$, $h_L/E_F = h_R/E_F = 0.1$, $h_M/E_F = 0.17$ and $\phi = 0$.

$\rangle + |\downarrow\uparrow\rangle)\sin(Q \cdot R)$. This state in F_R layer can be converted as $|\uparrow\downarrow\rangle e^{i(-Q' \cdot R' + \pi)} - |\downarrow\uparrow\rangle e^{-i(-Q' \cdot R' + \pi)} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\cos(Q' \cdot R' + \pi) + i(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)\sin(Q' \cdot R')$. We can see the spin singlet pair has a phase shift π in the

expression of the F_L layer and F_R layer. But the spin triplet pair with zero spin projection shows the same description in these two layers. These inferences are in agreement with our numerical results. As demonstrated in figure 4(d), f_3 shows a antisymmetric configuration around the middle F_M region. As a result, the superposition of two f_3 stemming from the left and right S will make their amplitudes cancel each other.

In conclusion, we have studied numerically the long-range supercurrent in a SFS structure including a non-collinear magnetic domain in the ferromagnetic region. We find the magnetic domain could induce a spin-flip process, which can reverse the spin orientations of the singlet Cooper pair when this pair propagate through the magnetic domain region. This process will make the singlet Cooper pair generate a phase-cancellation effect and acquire an additional π phase shift. If the ferromagnetic layers on both sides of magnetic domain have the same features (such as thickness and exchange field), the net change in the relative phase of the singlet Cooper pair is π when the pair passes through the entire F region. In this case, the long-range supercurrent mostly stems from the singlet Cooper pairs. The reason is that the equal spin triplet pairs are only present in the magnetic domain region and can not spread to the other two ferromagnetic layers. It is hoped that our results could propose a new way to generate the long-range Josephson current.

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